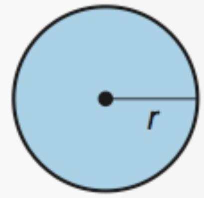
Geometry 11.5 Notes: Areas of Circles & Sectors (pp 691-4)

THEOREM

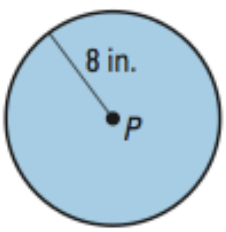
THEOREM 11.7 ***Area of a Circle***

The area of a circle is π times the square of the radius, or $A = \pi r^2$.

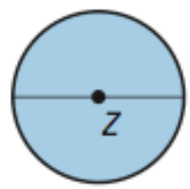


EXAMPLE 1 *Using the Area of a Circle*

a. Find the area of $\odot P$.



b. Find the diameter of $\odot Z$.



Area of $\odot Z$ = 96 cm^2

SOLUTION

a. Use $r = 8$ in the area formula.

$$A = \pi r^2$$
$$= \pi \cdot 8^2$$
$$= 64\pi$$
$$\approx 201.06$$

▶ So, the area is 64π, or about 201.06, square inches.

b. The diameter is twice the radius.

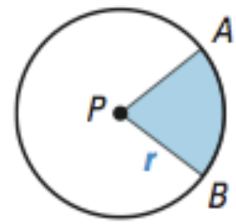
$$A = \pi r^2$$
$$96 = \pi r^2$$
$$\frac{96}{\pi} = r^2$$
$$30.56 \approx r^2$$
$$5.53 \approx r \qquad \text{Find the square roots.}$$

▶ The diameter of the circle is about 2(5.53), or about 11.06, centimeters.

Geometry 11.5 Notes: Areas of Circles & Sectors (pp 691-4)

1. **Guided Practice:** Find the area of a circle whose diameter is 30 in.

A **sector of a circle** is the region bounded by two radii of the circle and their intercepted arc. In the diagram, sector APB is bounded by $\overline{AP}$, $\overline{BP}$, and $\widehat{AB}$. The following theorem gives a method for finding the area of a sector.



THEOREM

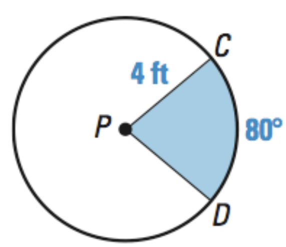
THEOREM 11.8 *Area of a Sector*

The ratio of the area A of a sector of a circle to the area of the circle is equal to the ratio of the measure of the intercepted arc to 360°.

$$\frac{A}{\pi r^2} = \frac{m\widehat{AB}}{360^\circ}, \text{ or } A = \frac{m\widehat{AB}}{360^\circ} \cdot \pi r^2$$

EXAMPLE 2 *Finding the Area of a Sector*

Find the area of the sector shown at the right.



SOLUTION

Sector CPD intercepts an arc whose measure is 80°. The radius is 4 feet.

$$A = \frac{m\widehat{CD}}{360^\circ} \cdot \pi r^2$$ Write the formula for the area of a sector.

$$= \frac{80^\circ}{360^\circ} \cdot \pi \cdot 4^2$$ Substitute known values.

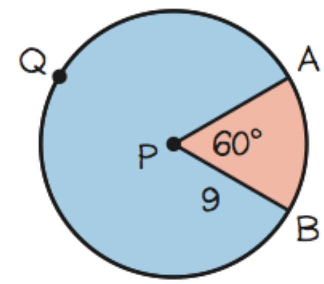
$$\approx 11.17$$ Use a calculator.

So, the area of the sector is about 11.17 square feet.

EXAMPLE 3 Finding the Area of a Sector

A and B are two points on a $\odot P$ with radius 9 inches and $m\angle APB = 60°$. Find the areas of the sectors formed by $\angle APB$.

SOLUTION



Draw a diagram of $\odot P$ and $\angle APB$. Shade the sectors.

Label a point Q on the major arc.

Find the measures of the minor and major arcs.

Because $m\angle APB = 60°$, $m\overparen{AB} = 60°$ and $m\overparen{AQB} = 360° - 60° = 300°$.

Use the formula for the area of a sector.

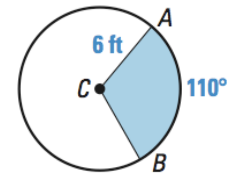
$$\text{Area of small sector} = \frac{60°}{360°} \cdot \pi \cdot 9^2 = \frac{1}{6} \cdot \pi \cdot 81 \approx 42.41 \text{ square inches}$$

$$\text{Area of larger sector} = \frac{300°}{360°} \cdot \pi \cdot 9^2 = \frac{5}{6} \cdot \pi \cdot 81 \approx 212.06 \text{ square inches}$$

2. **Guided Practice:** S and R are two points on $\odot W$ with radius 5 m and $m\measuredangle SWR = 45°$. Find the areas of the sectors formed by $\measuredangle SWR$.

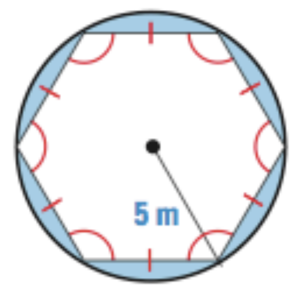
Geometry 11.5 Notes: Areas of Circles & Sectors (pp 691-4)

3. **Guided Practice:** Find the area of the shaded region:



EXAMPLE 4 *Finding the Area of a Region*

Find the area of the shaded region shown at the right.



SOLUTION

The diagram shows a regular hexagon inscribed in a circle with radius 5 meters. The shaded region is the part of the circle that is outside of the hexagon.

$$\boxed{\text{Area of shaded region}} = \boxed{\text{Area of circle}} - \boxed{\text{Area of hexagon}}$$

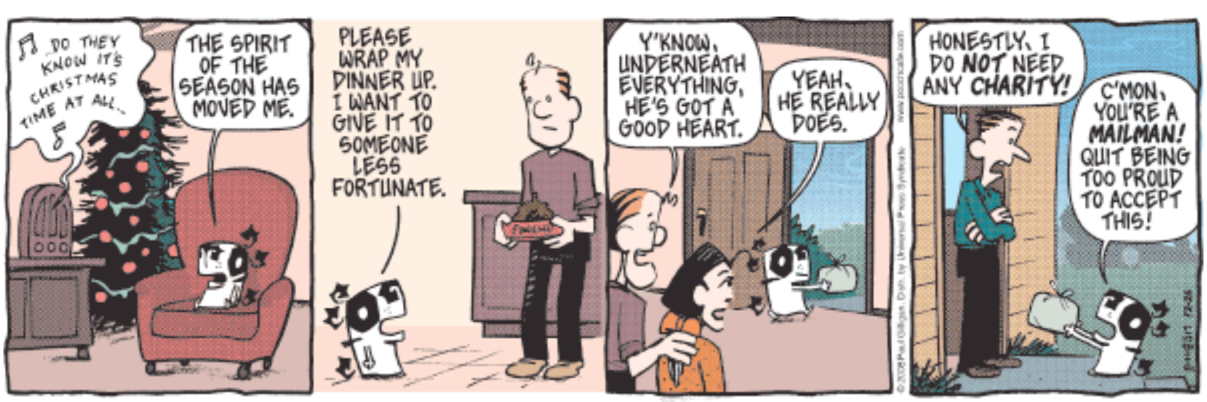
$$= \pi r^2 - \frac{1}{2}aP$$

$$= \pi \cdot 5^2 - \frac{1}{2} \cdot \left(\frac{5}{2}\sqrt{3}\right) \cdot (6 \cdot 5)$$

The apothem of a hexagon is $\frac{1}{2}$ • side length • $\sqrt{3}$.

$$= 25\pi - \frac{75}{2}\sqrt{3}$$

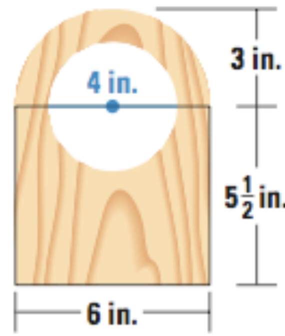
▶ So, the area of the shaded region is $25\pi - \frac{75}{2}\sqrt{3}$, or about 13.59 square meters.



Geometry 11.5 Notes: Areas of Circles & Sectors (pp 691-4)

EXAMPLE 5 ***Finding the Area of a Region***

WOODWORKING You are cutting the front face of a clock out of wood, as shown in the diagram. What is the area of the front of the case?



SOLUTION

The front of the case is formed by a rectangle and a sector, with a circle removed. Note that the intercepted arc of the sector is a semicircle.

$$\text{Area} = \boxed{\text{Area of rectangle}} + \boxed{\text{Area of sector}} - \boxed{\text{Area of circle}}$$

$$= 6 \cdot \frac{11}{2} + \frac{180^\circ}{360^\circ} \cdot \pi \cdot 3^2 - \pi \cdot \left(\frac{1}{2} \cdot 4\right)^2$$

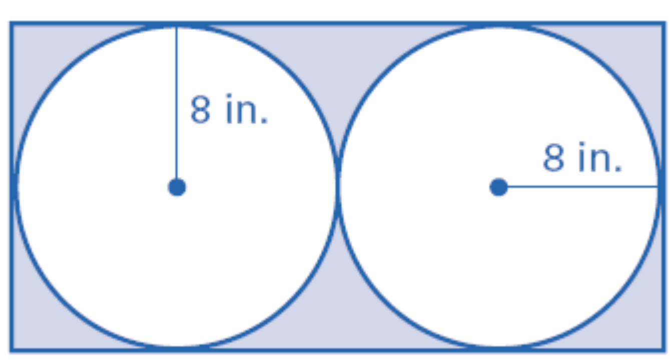
$$= 33 + \frac{1}{2} \cdot \pi \cdot 9 - \pi \cdot (2)^2$$

$$= 33 + \frac{9}{2}\pi - 4\pi$$

$$\approx 34.57$$

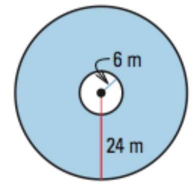
The area of the front of the case is about 34.57 square inches.

4. Guided Practice: Find the area of the shaded region.

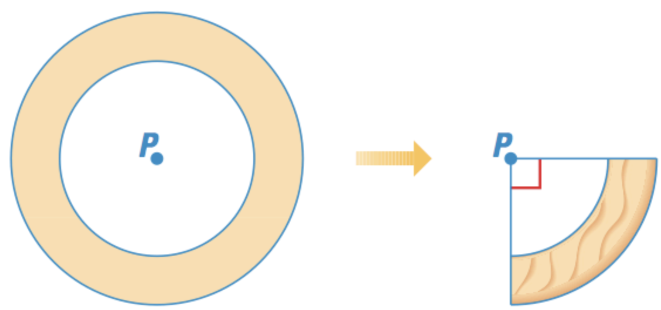


Geometry 11.5 Notes: Areas of Circles & Sectors (pp 691-4)

5. Guided Practice: Find the area of the shaded region:



Complicated shapes may involve a number of regions. In Example 6, the curved region is a portion of a ring whose edges are formed by concentric circles. Notice that the area of a portion of the ring is the difference of the areas of two sectors.



Geometry 11.5 Notes: Areas of Circles & Sectors (pp 691-4)

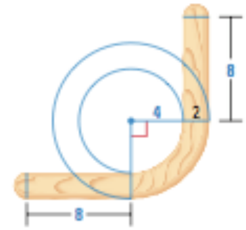
FOCUS ON APPLICATIONS

BOOMERANGS are slightly curved at the ends and travel in an arc when thrown. Small boomerangs used for sport make a full circle and return to the thrower.

PROBLEM SOLVING STRATEGY

EXAMPLE 6 *Finding the Area of a Boomerang*

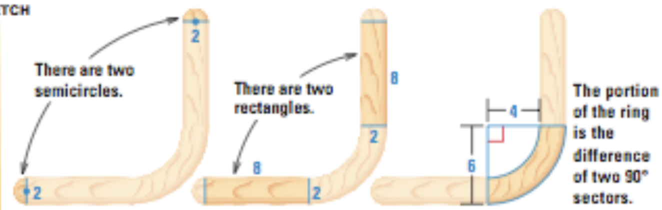
BOOMERANGS Find the area of the boomerang shown. The dimensions are given in inches. Give your answer in terms of π and to two decimal places.



SOLUTION

Separate the boomerang into different regions. The regions are two semicircles (at the ends), two rectangles, and a portion of a ring. Find the area of each region and add these areas together.

DRAW AND LABEL A SKETCH Draw and label a sketch of each region in the boomerang.



VERBAL MODEL $\boxed{\text{Area of boomerang}} = 2 \cdot \boxed{\text{Area of semicircle}} + 2 \cdot \boxed{\text{Area of rectangle}} + \boxed{\text{Area of portion of ring}}$

LABELS

Area of semicircle $= \frac{1}{2} \cdot \pi \cdot 1^2$ (square inches)

Area of rectangle $= 8 \cdot 2$ (square inches)

Area of portion of ring $= \frac{1}{4} \cdot \pi \cdot 6^2 - \frac{1}{4} \cdot \pi \cdot 4^2$ (square inches)

REASONING

$$\boxed{\text{Area of boomerang}} = 2\left(\frac{1}{2} \cdot \pi \cdot 1^2\right) + 2(8 \cdot 2) + \left(\frac{1}{4} \cdot \pi \cdot 6^2 - \frac{1}{4} \cdot \pi \cdot 4^2\right)$$
$$= 2\left(\frac{1}{2} \cdot \pi \cdot 1\right) + 2 \cdot 16 + \left(\frac{1}{4} \cdot \pi \cdot 36 - \frac{1}{4} \cdot \pi \cdot 16\right)$$
$$= \pi + 32 + (9\pi - 4\pi)$$
$$= 6\pi + 32$$

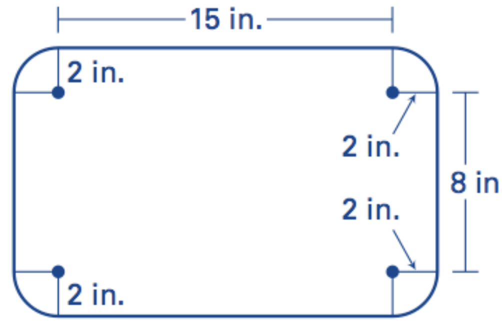
▶ So, the area of the boomerang is $(6\pi + 32)$, or about 50.85 square inches.

off the mark.com by Mark Parisi

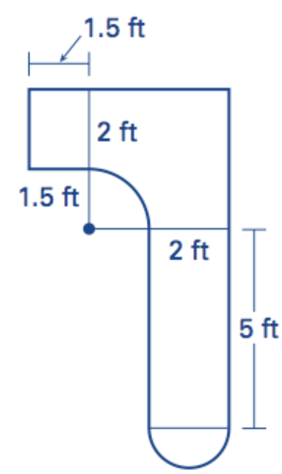
Geometry 11.5 Notes: Areas of Circles & Sectors (pp 691-4)

Guided Practice.

6. A hole is to be cut out of a countertop to accommodate the sink shown below. What is the area of the countertop that needs to be cut out to accommodate the sink if the sink fits exactly in the hole?



12. Find the area of the countertop.



12. Describe how to find the area of a sector of a circle.

13. Describe the boundaries of a sector of a circle.